

Name: Solutions / Answers

Answer each question without a calculator.

1. Find the points of continuity and the points of discontinuity of the function

$$f(x) = \frac{\sin x}{x}$$

$f(x)$ is continuous for all real x such that $x \neq 0$

2. Find the points of continuity and the points of discontinuity of the function

$$f(x) = \frac{\cos x}{x^2 + 1}$$

$f(x)$ is continuous for all real x

3. Find the points of continuity and the points of discontinuity of the function

$$f(x) = \frac{\cos x}{x^2 - 1}$$

$f(x)$ is continuous for all real x such that $x \neq -1$ and $x \neq 1$

4. Find the points of continuity and the points of discontinuity of the function

$$f(x) = \frac{x^2 + 5x - 24}{x^2 - 12x + 27} = \frac{(x+8)(x-3)}{(x-3)(x-9)}$$

$f(x)$ is continuous for all real x such that $x \neq 3$ and $x \neq 9$

5. Write an extended function for the function $f(x) = \frac{\sin x}{x}$ that is continuous at the point $x = 0$

$$g(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

6. Write an extended function for the function $f(x) = \frac{x^3 - 7x - 6}{x^2 - 9}$ that is continuous at the point $x = 3$

$$f(x) = \frac{(x-3)(x+2)(x+1)}{(x-3)(x+3)}$$

$$g(x) = \frac{x^2 + 3x + 2}{x + 3}$$

$x = 3$

$$\begin{array}{r} 3 \overline{) 1 \ 0 \ -7 \ -6} \\ \underline{3 \ 9 \ 6} \\ 1 \ 3 \ 2 \ 0 = R \\ x^2 + 3x + 2 \\ (x+1)(x+2) \end{array}$$

7. Write an extended function for the function $f(x) = \frac{x-2}{x^3-8}$ that is continuous at the point $x=2$

$$f(x) = \frac{x-2}{(x-2)(x^2+2x+4)}$$

$$g(x) = \frac{1}{x^2+2x+4}$$

8. Write an extended function for the function $f(x) = \frac{x-4}{\sqrt{x}-2}$ that is continuous at the point $x=4$

$$f(x) = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}$$

$$g(x) = \sqrt{x} + 2$$

9. Is the function $f(x) = \frac{1}{x}$ continuous on its domain?

Yes

10. Is the function $f(x) = \frac{|x|}{x}$ continuous on its domain?

Yes

11. Is the function $f(x) = \begin{cases} \frac{x^2+9}{x-3} & \text{if } x \neq 3 \\ 7 & \text{if } x = 3 \end{cases}$ continuous on its domain?

No

12. True or False A continuous function cannot have a point of discontinuity. Justify your answer.

See page 81. A continuous function is one that is continuous at every point of its domain. A continuous function need not be continuous on every interval. For example, $y = \frac{1}{x}$ is not continuous on $[-1, 1]$, but it is a continuous function on its domain.

13. True or False: It is possible to extend the definition of a function f at a jump discontinuity $x=a$ so that f is continuous at $x=a$. Justify your answer.

False, for example the function $f(x) = \frac{|x|}{x}$

has a jump discontinuity at $x=0$.

$\lim_{x \rightarrow 0^-} f(x) = -1$ and $\lim_{x \rightarrow 0^+} f(x) = 1$ and there is no way

to bridge the gap that occurs at $x=0$.

14. Find each point of discontinuity for the function $f(x) = \frac{x^2 - 9}{x^2 - 3x} = \frac{(x-3)(x+3)}{x(x-3)}$

Which of the discontinuities are removable? Which are not removable?

$f(x)$ has an asymptotic discontinuity at $x=0$,
and $f(x)$ has a removable discontinuity at $x=3$.

15. Find each point of discontinuity for the function $f(x) = \begin{cases} \frac{x-2}{x-2}, & x \neq 2, x \neq 4 \\ 3, & x = 2 \\ -2, & x = 4 \end{cases}$

Which of the discontinuities are removable? Which are not removable?

$f(x)$ has a removable discontinuity at $x=4$
and a jump discontinuity at $x=2$.

16. Find a value for "a" for which the function $f(x) = \begin{cases} 2x+3, & x \leq 2 \\ ax+2, & x > 2 \end{cases}$ is continuous.

$$2(2)+3 = a(2)+2$$

$$7 = 2a + 2$$

$$5 = 2a \quad a = 2\frac{1}{2}$$

17. Find a value for "a" for which the function $f(x) = \begin{cases} 4-x^2, & x < -1 \\ ax^2-1, & x \geq -1 \end{cases}$ is continuous.

$$4 - (-1)^2 = a(-1)^2 - 1$$

$$4 - 1 = a - 1$$

$$4 = a$$

18. Which of the following points of discontinuity of $f(x) = \frac{x(x-2)^3(x+1)}{x(x-2)^2(x+1)^2}$ is not removable?

a. $x=0$

b. $x=2$

☒ c. $x=-1$

d. $x=-2$

e. $x=1$

19. On which of the following intervals is $f(x) = \frac{1}{x^2 - 2}$ not continuous?

a. $(2, \infty)$

b. $(-\infty, -2)$

c. $(-1, 1)$

☒ d. $(1, 2)$

e. $[2, 3)$

20. On which of the following intervals is $f(x) = \frac{1}{\sqrt{x}}$ not continuous?

a. $(0, \infty)$

☒ b. $[0, \infty)$

c. $(0, 2)$

d. $(1, 2)$

e. $[1, \infty)$